Non-Gaussian conductance noise in disordered electronic systems due to a non-linear mechanism

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Abstract

We present results of conductance-noise experiments on disordered films of crystalline indium oxide with lateral dimensions $2\mu m$ to $1mm$. The power-spectrum of the noise has the usual $1/f$ form, and its magnitude increases with inverse sample-volume down to sample size of $2\mu m$, a behavior consistent with un-correlated fluctuators. A colored second spectrum is only occasionally encountered (in samples smaller than $40\mu m$), and the lack of systematic dependence of non-Gaussianity on sample parameters persisted down to the smallest samples studied ($2\mu m$). Moreover, it turns out that the degree of non-Gaussianity exhibits a non-trivial dependence on the bias V used in the measurements; it initially increases with V then, when the bias is deeper into the non-linear transport regime it decreases with V. We describe a model that reproduces the main observed features and argue that such a behavior arises from a non-linear effect inherent to electronic transport in a hopping system and should be observed whether or not the system is glassy.

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I. INTRODUCTION

Conduction noise is a property of essentially all electronic systems. A common form of this noise has a $1/f^\eta$ power-spectrum with η of order unity [1]. Such a spectrum may result from the superposition of many fluctuators with individual frequencies ω_i extending uniformly over the observed range [1]. When these fluctuators are independent, the central limit theorem mandates that the power-spectrum be Gaussian, and this is presumably the generic result in the thermodynamic limit. Therefore when a system with power-spectrum of the $1/f$ type and a non-Gaussian nature is encountered, it seems reasonable to conclude that the associated fluctuators are correlated.

Identifying the underlying source of the correlations however, is not a trivial task as there could be several reasons for non-Gaussian noise, including artifacts, and correlations in the noise may appear even when the fluctuators are basically independent. For example, Seidler *et al* [2], using their dynamical-current-redistribution (DCR) model, argued that a non-Gaussian noise may result from features that are peculiar to the process of electrical conductivity in a network; Their considerations are especially relevant for inhomogeneous systems, and when the fluctuations are strong. Both ingredients are inherent to transport in disordered conductors where non-Gaussianity were frequently observed [3]. The archtypical system of this class is a hopping system where transport is confined to a percolation network. Such a system may be viewed as a random-resistor-network in which each resistor r_{ij} is associated with pair of sites i, j that are connected by a hopping process [4]. The wide distribution of the r_{ij}'s in the random-resistor-network leads to several unique features of electronic transport in such a medium. The most familiar feature is that the current in the system is carried by a percolation-network [5, 6, 7] involving a relatively small number of 'critical' resistors. This percolation problem differs from the classical, 'geometric' scenario in two essential accounts; First, the current-carrying network (CCN) of the hopping system is temperature dependent - it becomes progressively more rarefied as temperature decreases. Secondly, the resistances that comprise the CCN, as well as elemental fluctuators that modulate them, typically exhibit non-linear effects. At low temperatures, these non-linear effects are quite prominent even for small applied fields. As we shall see, these features introduce a new set of considerations into the question of noise correlations.

In this paper we report on a study of noise statistics of crystalline indium-oxide films

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(In_2O_{3-x}) with their disorder tuned to make them strongly localized. Previous studies of $1/f$ noise using In_2O_{3-x} films has focused on the *metallic* regime and employed macroscopic samples [8]. The emphasis in the current study is on the results obtained with samples that exhibit prominent mesoscopic effects. The other aspect that distinguishes this work from the previous study (where room temperature measurements were employed) is that the noise experiments were carried out at liquid helium temperatures. The original motivation for these experiments was an attempt to find the spatial extent of the correlation length in the glassy phase of In_2O_{3-x} films by monitoring the noise characteristics of such films, in particular, the second-spectrum [9], as function of their size. The rationale being that below a certain sample size the correlated nature of the electron glass should manifest itself in a respective correlation of the conductance fluctuations. That should presumably occur once the spatial scale associated with the glassy effects exceed the sample size. Samples with lateral dimensions ranging from 1mm down to $2\mu m$ were studied for this purpose. A colored second spectrum, indicative of correlations, was indeed found for samples with sizes at the lower part of this range. However, the phenomenology encountered in these measurements was not in line with the expectation based on the above scenario. In particular, even in the small size regime, the occurrence of colored second spectrum was not consistent; samples with nearly identical parameters gave conflicting results. This led us to suspect that the source of correlations is not related to the underlying glass. Further experiments revealed that that the degree of coloration in the second spectrum depends in a non trivial way on the bias V used in the measurements. We propose a heuristic picture that qualitatively accounts for these observations. This model is generic to transport in disordered systems where transport is inherently inhomogeneous, and does not depend on the system being glassy.

II. SAMPLES PREPARATION AND MEASUREMENTS TECHNIQUES

The In_2O_{3-x} films used here were e-gun evaporated on either a $110\mu m$ thick microscope-cover glass, or on a SiO_2 insulating layer ($0.5\mu m$ thick) thermally grown on a Si wafer. The latter was boron doped and had resistivity $2 \cdot 10^{-3}\Omega cm$, deep in the degenerate regime. It thus could be used as the gate electrode for a low-temperature measurement with the sample configured as a MOSFET device in which a thin film of In_2O_{3-x} served as the active layer.

Lateral dimensions of the samples were controlled using a stainless steel mask (for samples larger than $0.5mm$), or optical lithography (for sizes in the range $30-200\mu m$) and e-beam lithography for samples smaller than $30\mu m$. Samples used in this study had length (L) and width (W) that ranged from $2mm$ down to $2\mu m$ and typical thickness $d = 55 \pm 5\overset{\circ}{A}$. The 'source' and 'drain' contacts were made from thermally evaporated $\approx 500\overset{\circ}{A}$ thick gold films. Fuller details of sample preparation and characterization are given elsewhere [10].

Conductance measurements were performed using two terminal ac technique, employing ITHACO-1211 current preamplifier and PAR-124A lock-in amplifier. In the MOSFET-like samples, gate voltage sweeps were affected by charging a $10\mu F$ capacitor with a constant current source (Keithley 220). All the measurements reported here were performed with the samples immersed in liquid helium at $T = 4.11$K held by a 100 liters storage-dewar. This allowed long term measurements without disturbing the samples as well as a convenient way to maintain a stable bath temperature. These requirements are of particular importance for studies of glassy systems where sample history may influence time dependent measurements, as was demonstrated in previous studies using such samples [10]. To cater for this, samples were allowed to equilibrate for at least 15 hours prior to any conductance versus time measurement. No change in the nature of the noise of a given sample was found when measurements were repeated a week after the initial cool-down.

A typical hopping length at these temperatures, and for the range of resistances used in this work is $\approx 200\overset{\circ}{A}$ [11], which makes such samples effectively two-dimensional (2D). This fact will be used in the proposed model described in section IIIC.

Noise measurements employed a two terminal technique. These were performed by biasing the sample with a constant (dc) voltage source (home-made rechargeable battery-stack) while measuring the resulting current fluctuations by the ac voltage drop V_{ac} detected across a series resistor. This V_{ac} was monitored by a Dynamic Signal Analyzer (HP35670A) buffered by EG&G5113 low-noise pre-amplifier. These data were then used to calculate the first and second power-spectrum. The latter were implemented by the method suggested by Restle *et al* [12]. As will be demonstrated below, the noise magnitude of the samples studied here is quite large. This is due to three reasons; (1) The samples reported here are strongly disordered and exhibit large noise even when of macroscopic size [8]. (2) The samples are physically small, c.f., figure 2. (3) At the measured temperature, the samples are rather deep into the hopping regime, which means that the effective volume for the current carrying

process in these samples may be considerably smaller than their geometrical volume. These factors combine to give a large noise magnitude, which made it unnecessary to use a more comprehensive measurement configuration such as a 5-probe technique (that is awkward to employ for insulating samples being inherently inhomogeneous). We did however check that the contacts do not contribute to the noise by measuring few samples using a 4-probe technique.

The stability of the temperature bath afforded by the 100 liter storage dewar was sufficiently accurate to neglect time-dependent temperature effects on the samples conductance relative to the inherent 1/f noise. This was ascertained by monitoring the resistance fluctuations of a Ge thermometer attached to the sample stage as in Fig.5 of Vaknin et al [10].

III. RESULTS AND DISCUSSION

A. Preliminary measurements

A conductance time-trace and associated power-spectrum for the smallest sample in the series studied are shown in Fig.1. Apart from its large magnitude (presumably due to the small sample size), the noise has similar characteristics as these of metallic samples of this material [8], in particular the overall shape of the power-spectrum retains its $1/f^\eta$ character although some deviations from a power-law may be observed; a best-fit to the data yields $\eta \simeq 1.1$ but the low f region seems to follow a faster dependence. This is not surprising as such samples contain a rather small number of fluctuating resistances as will be shown later. The dependence of the noise magnitude on sample volume is shown in Fig.2 for several of the studied samples. To facilitate comparison, the figure includes only samples that are all of the same thickness, and have similar resistances. Note that the noise magnitude is inversely proportional to volume down to sample size of $2x2\mu m$. This result is consistent with what is expected of ensemble-averaged un-correlated fluctuators. It might therefore suggest that, if there are correlations in the noise in this system, they occur on scales smaller than $2\mu m$. Surprisingly then, when analyzing preliminary data for samples in this series we encountered several traces that yielded a frequency-dependent second-spectrum. Such occurrences were not encountered in samples with sizes $\geq 100\mu m$, and they were still rare in the range of

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sizes $30 - 50\mu m$. Colored second-spectrum could be observed in samples as large as $\approx 40\mu m$ but in unpredictable way; Two samples with nearly identical parameters (resistance, size), and measured under similar conditions, gave conflicting results; one exhibited frequency-dependent second spectrum, while the other had a Gaussian spectrum. An example of such a 'conflicting couple' is shown in Fig.3 for $30x40\mu m$ samples. This lack of systematic behavior persisted down to our smallest sample sizes, although the frequency of a colored second-spectrum appearances seem to grow with decreasing size.

In contrast with the inconsistent appearance of noise correlations, all samples in this series systematically show the same glassy features as macroscopic samples with similar parameters. For example, Fig.4 shows the 'memory' cusp, which is the earmark of the electron-glass [10], for the $30x40\mu m$ sample shown in Fig.3. Note that this actually is the sample that exhibits *Gaussian* noise. It is therefore unlikely that the f-dependent second spectrum can be related to the correlations due to the interactions that are associated with electron-glass dynamics. This conclusion will be re-enforced by a more elaborate analysis and the ensuing discussion below.

In addition to the glassy cusp, the data in Fig.4 show mesoscopic conductance fluctuations (CF). These are the 'fingerprints' of the underlying CCN, and reflect the process by which some critical resistors in the CCN are replaced by other critical resistors as the chemical potential is varied [13]. The relative magnitude of the CF is a function of $\mathcal{L}$, the correlation length of the percolation network. This is based on two assumptions: First, the basic conductance swing ΔG associated with a critical resistor is of the order of its conductance G. This is a characteristic feature of the strongly localized regime, (which for the present case of 2D samples means sheet resistance $R_\square$ that fulfills $R_\square \gg \frac{\hbar}{e^2}$) [14]. Secondly, the relative fluctuation amplitude $\Delta G/G$ for the entire sample is essentially determined by the square root of the number of critical resistors in the sample $N \approx \frac{LW}{\mathcal{L}^2}$, therefore, $\frac{\Delta G}{G} \approx \sqrt{\frac{\mathcal{L}^2}{LW}}$. This relation will be used in this paper to estimate $\mathcal{L}$ as well as it dependence on applied voltage.

B. Dependence on applied field

Low temperature transport measurements on hopping systems are very sensitive to the value of the voltage V used. It is notoriously difficult to achieve linear response conditions in

such cases, especially for small samples at low temperatures, and deviations from Ohm's law are hard to avoid. This is illustrated in Fig.5 for the two samples that will be discussed in this subsection. Usually, the effect of a not-small-enough-voltage on the measurement is in the same direction of raising the sample temperature. For example, the resistance decreases with V (Fig.5), and so does the noise magnitude (figure 7c), in both cases, the effect is monotonous with V. It turns out that the degree of non-Gaussianity of the type considered here, behaves in a qualitatively different way. In particular, it is non-monotonous with V, and it tends to vanish at both, high bias *and* at low bias thus peaking at an intermediate value of bias.

As a quantitative measure of the degree of non-Gaussianity we take the area under the curve of the second spectrum as depicted in Fig.3, which we label as $Int(S_2)$. Using this scheme, we plot the dependence of this quantity on V for one of our smallest sample that exhibits non-Gaussianity (Fig.6). The first set of data (solid circles) were taken initially without regard to the order of changing V just to test the effect produced. When it was realized that the correlations may disappear at high V, the field was taken to a much larger value ($V = 46$ mV) to better define the asymptotic behavior of $Int(S_2)$. A later attempt to add more points to the curve failed to reproduce the position of the peak in $Int(S_2)$ versus V obtained in the first series. Rather, the curve seems to have shifted towards a lower bias. The average sample resistance r and its dependence on voltage $r(V)$ were not affected by the high V exposure. We shall return to this 'mesoscopic' effect following the discussion in the next section.

Figure 7 shows a more elaborate study of another $2x2\mu m$ sample with similar noise characteristics except that now care was taken not to subject the sample to an excessively large V. The same pattern in terms of $Int(S_2)$ versus V emerged (upper graph in Fig.7), and this time the curve was fairly well reproduced by a second series of measurements. Along with the second spectrum analysis, the figure shows the respective dependence of the power-spectrum parameters η and amplitude. Neither shows the non-monotonic V dependence exhibited by $Int(S_2)$. It is natural that above a certain field, both $Int(S_2)$ and the noise magnitude decrease with V; a large field, like temperature, decreases the hopping-length and $\mathcal{L}$, which in turn means larger number of fluctuators to average over. The tale-telling result is that, below certain field, $Int(S_2)$ diminishes when V decreases in a way that suggests a much smaller effect as $V \rightarrow 0$. In other words, the noise appears to be correlated *because*

the measurement is not in the linear response regime.

A plausible scenario that leads to correlations between individual fluctuators is based on two ingredients; A) the current in the system is carried by a percolation-network [5, 6, 7], and B) the (average) frequency ω_i of a fluctuator i is, among other things, a function of the local voltage V_i. The latter is true for a generic two-level-state whether of 'atomic' or 'electronic' nature; The frequency of the two-level-state usually depends exponentially on the local voltage [15], so this is a sensitive source of inter-modulation once the local V_i's are re-distributed by the DCR effect. Ingredient A is an inherent feature of variable-range-hopping systems, and it is probably a common feature in other disordered conductors as well. The system may then be viewed as a random-resistor-network where the active fluctuators are part of it (or situated nearby such that can modulate a resistance that is in the current path). When a fluctuator j in the network, changes its state, the local voltage on fluctuator i will change too due to the continuity of the current carrying network. If the resulting voltage change δV_i is not much smaller than $k_B T$, the switch of j will result is a change $\delta\omega_i$ of fluctuator i frequency. Such an effect is the basic building-block of a hierarchical correlation chain where the slower fluctuator modulates a faster one. Naturally, this mechanism for non-Gaussianity must vanish with the applied field as indeed is observed. It should also be negligible when the system size is much bigger than the range of the proposed inter-modulation effect. These considerations will be now dealt with in a more formal way, and the results will be compared with the experiments.

C. Theoretical considerations and comparison with experiments

We consider a 2D hopping system and use the standard percolation scheme [16]. Focusing attention on a given critical resistor j which is affected by some fluctuator. For simplicity we assume that each resistor is coupled only to a single fluctuator and will denote this fluctuator by the same index j. Two different candidates for the role of the basic fluctuators may be considered. The first is an 'atomic' two level system as in amorphous materials (see, e.g., Mott and Davis [17]). The second one is an aggregate composed of localized sites (not necessarily part of the percolation cluster), and having two metastable configurations characterized by different distribution of the electrons over the sites. The simplest object of such a sort is a pair of hopping sites occupied by a single electron considered as a source

of 1/f noise in [18]. Although quantitatively the effects of atomic and electronic two level systems are expected to be different and depend on different parameters of the material, in both cases the fluctuators are expected to be sensitive to the local electric fields (for the atomic two level systems the corresponding coupling is related to a presence of electric dipolar moment of the atomic two level system). The model considerations given below are applicable for either mechanism, although the microscopic equations for the coupling coefficients are naturally different.

The fluctuation of the corresponding voltage swing, δV_j, will inevitably lead to a fluctuation of the voltage across all other resistors. This is the DCR effect considered by Seidler et al [2] and shown to lead to a colored second-spectrum of magnitude $S_{2,DCR}$. Next we show that in the hopping system the voltage re-distribution in the CCN results in a more elaborate coupling mechanism between different fluctuators. In particular, this results in a dependence of the switching rate of the fluctuator i on the state of the fluctuator j. This coupling gives an additional, non-linear, contribution to the frequency dependent second spectrum ($S_{2,corr}$). At large bias both $S_{2,DCR}$ and $S_{2,corr}$ are suppressed with V due to an increase of the number of effective fluctuators N. We will show that $S_{2,corr}$ initially increases with V and peaks at some intermediate V consistent with the experimentally observed behavior (figures 6 and 7).

Denoting by δV_{ji} the voltage swing on resistor-i affected by the corresponding variation of resistor-j, and assume that the main ensuing effect is a variation of Δ_i, the difference in energy between the two states of the fluctuator;

$$\delta(\Delta_i) = B_i \delta V_{ji} \tag{1}$$

where B_i is a coupling coefficient. In terms of the correlation length $\mathcal{L}$, and for $R_{ij} \gg \mathcal{L}$ (R_{ij} is the distance between the critical resistors i and j) δV_{ji} can be estimated as:

$$\delta V_{ji} \sim V_j \frac{\delta G_j}{G_j} \frac{\mathcal{L}}{R_{ij}} \tag{2}$$

where in the regime $eV_j << k_B T$ one has $|\delta G_j/G_j| = |\delta \varepsilon_j/k_B T|$, and $\delta \varepsilon_j$ is related to the modulation of the resistor activation energy by the current re-distribution.

The variation of Δ_j leads to a variation of the fluctuator dwell times. The latter can be written as [19]

$$\tau^{-1} = \tau_+^{-1} + \tau_-^{-1} = \tau_+^{-1}\left(1 + \frac{n}{1-n}\right) = \tau_+^{-1}(1-n)^{-1} \tag{3}$$

where τ_+ and τ_- are the dwell times for the upper and lower level, respectively, and n is the occupation at the upper-level state. Note that τ_+ corresponds to a transition accompanied by the emission of the phonon with a frequency $\omega = \Delta/\hbar$. Thus, the change in τ_+ due to the fluctuation of Δ is

$$\delta\tau_+ \simeq -\tau_+ \frac{\alpha}{\Delta}(\delta\Delta) \tag{4}$$

where we have taken into account that $\tau_+^{-1} \propto \Delta^\alpha$. Note that $\alpha \approx 3$ for either a fluctuator of electronic nature in the limit of small Δ_i [19], as well as for a fluctuator of atomic nature [20],

Combining eqs. 3,4 we obtain for the fluctuation of the relaxation time of i-th fluctuator:

$$\delta\tau_i = -\tau_i \left(\frac{\alpha}{\Delta_i} - \frac{n_i}{k_B T} \right) \delta\Delta_i \tag{5}$$

Note that for an ideal $1/f$ first spectrum (which, strictly speaking, can be realized only for $L \rightarrow \infty$), the fluctuations of τ_i would not lead to a variation of S_1. However for a small sample size (where deviations from $1/f$ spectrum may be observable as, e.g., Fig.1), the first spectrum may be significantly affected by fluctuations of τ_i. In the limit $eV_j << k_B T$ (where $\mathcal{L} = \mathcal{L}_0$) the effect is proportional to V_j and thus to the total bias V ($V_j \approx \frac{V\mathcal{L}}{L}$).

It can be shown (see Appendix 1) the resulting contribution to S_2 can be estimated as

$$S_{2,corr} \propto \alpha^2 B_i \delta V_{ij} \propto V \tag{6}$$

By comparison, the contribution of the DCR is given as

$$S_{2,DCR} \propto T\bar{\gamma}_{ij} \tag{7}$$

Here

$$\gamma_{ij} = \frac{\delta V_{ij}^2}{V_i^2}$$

which at $eV_i << k_B T$ does not depend on V while $\bar{\gamma}_{ij}$ means an average magnitude of γ_{ij}; the prefactors in eqs. 6,7 differ from one another only by a numerical coefficient of the order of unity.

Eqs.6,7 then suggest that $S_{2,corr} > S_{2,DCR}$ if

$$e\bar{V}_i > k_B T \frac{\bar{\gamma}_{ij}}{\bar{B}_i \alpha^2} \tag{8}$$

In other words, our mechanism dominates over the DCR even when $e\bar{V}_i \ll k_B T$ provided

$$\bar{\gamma}_{ij} < \alpha^2 \bar{B}_i \tag{9}$$

Recall that B_i describes a relative effect of the fluctuation of the resistor potential on the state of the nearby fluctuator while $\bar{\gamma}_{ij}$ describes the relative fluctuation of the voltage on the resistor i due to a fluctuator modulating resistor j. In general, $R_{ij} > \mathcal{L}$ and this ratio is small. At the same time the effect of the resistor voltage on the fluctuator can be large enough. It holds in particular for the fluctuators of the electronic origin if their size is comparable to the inter-site distance within the hopping resistor r_h provided that it is situated at the distances less or comparable to r_h. It also holds for the structural fluctuators provided they are situated close enough to the site with a lower energy. In these cases $B_i \sim 1$ therefore condition 9 may be obeyed.

Now let us discuss the regime $\bar{V}_i \geq k_B T$ when the hopping conductivity is strongly non-linear. It can be shown that the fluctuation of the conductance of resistor i resulting from the fluctuation of its activation energy for $eV_i > k_B T$ is still given by $dG_i \simeq -G_i(d\varepsilon_i)/k_B T$. However the relative fluctuation of the total conductance of the sample depends on the correlation length $\mathcal{L}$:

$$S_1(\omega) \equiv \frac{(dG, dG)_\omega}{G^2} = \frac{\mathcal{L}^4}{L^4} \sum_i \frac{(dG_i, dG_i)}{G_i^2} \tag{10}$$

where L is a linear size of the sample while $\mathcal{L}$ is the correlation length of the percolation cluster which in the nonlinear regime can be estimated as [21]

$$\mathcal{L} \simeq \mathcal{L}_0 \left(\frac{k_B T}{e E \mathcal{L}_0} \right)^{\nu/(1+\nu)} \tag{11}$$

where E is an average electric field within the sample, $\mathcal{L}_0 = \mathcal{L}(V \to 0)$, and ν is the percolation theory index (for 2D $\nu \sim 4/3$). Thus $\mathcal{L} \propto V^{-\nu/(1+\nu)} = V^{-4/7}$. Correspondingly, one has

$$S_1(\omega) \propto \frac{\mathcal{L}^2}{L^2} \propto V^{-8/7} \tag{12}$$

(we have taken into account that the result of a summation over the effective resistors is proportional to the number of these resistors).

This ensemble-averaging effect is expected to be even stronger on the second spectrum since it is a convolution of the two first spectra. So one expects $S_2 \propto N^{-2}$ where $N = (L/\mathcal{L})^2$

is the number of fluctuators, then:

$$S_{2,corr} \propto \frac{V}{N^2(V)} \qquad (13)$$

This is strictly obeyed for $eV < k_B T$, at higher voltages there may be contributions of other non-linear mechanism not considered here. Note that Eq.13 contains two factors. The numerator (V) depends on the coupling coefficient given in Eqs. 1-2 (see Appendix) and is indeed defined for the case $eV < k_B T$. For larger bias this coupling coefficient is suppressed due to the non-linearity of the medium and corresponds to a sub-linear behavior; we do not consider this effect in detail. The denominator depends on the statistical average discussed above Eq.13, and it remains the same for $eV > k_B T$ while its dependence on V follows from the considerations given in Eq.11.

To compare, the respective contribution of the DCR mechanism has the following dependence on V :

$$S_{2,DCR} \propto \frac{1}{N^2(V)} \qquad (14)$$

Let us now see how these expectations compare with our experiments. To find the qualitative dependence on V of the second-spectrum amplitude we need to know the function $N^2(V)$, namely how the number of fluctuators varies with V over the range relevant for the experiment. This may be estimated theoretically using similar considerations as those that led to eq.12 above. The dependence of the first spectrum amplitude on V is in rough agreement with this equation (c.f., Fig.7c). However, the data for the second spectrum amplitude were taken over more extensive range of V, exceeding the limits of validity of the power-law relation expected by eq.12. It is therefore necessary to get an estimate for $N^2(V)$ from experiments. That can be done through the use of data for the conductance fluctuation versus V such as the results shown in Fig.8a. To construct an empirical $N^2(V)$, one then uses the relation $\frac{\Delta G}{G}(V) \approx \sqrt{\frac{\mathcal{L}^2}{LW}} = N(V)^{-\frac{1}{2}}$ discussed in section 2. This procedure yields the $N(V)$ depicted in Fig.8b, which empirically, fits rather well an exponential dependence; $N(V) \propto \exp(\sqrt{V})$ [22]. Using this form in eqs.13,14 one gets the qualitative dependence on V for the two mechanisms for the second-spectrum considered here. This is schematically illustrated in Fig.9. The overall shape in this plot is in fair agreement with the experimental curves (c.f., Fig.6 and 7) although it seems that the small bias regime would fit better a faster than linear with V relation. To estimate the value of the voltage V_i (that at the peak of $Int(S_2)$ versus V should probably be of the order of $k_B T$, c.f., Fig.9), one needs to know

the number of critical resistors in the sample. As noted above, this can be done based on the relative magnitude of the CF, namely, the reproducible fluctuations in $G(V_g)$ generated by sweeping the gate voltage V_g. The number of critical resistors along the sample in Fig.6 can be be estimated using the data in Fig.8b. Note that the applied voltage at the peak of $Int(S_2)$ is $V_p \approx 5mV$ and $V_p \approx 10mV$ for the data in squares and circles respectively (c.f., Fig.6). The value N for $V \approx 10mV$ can be read from Fig.8b as $N \approx 260$, which then means that the value of V_i at the peak is $\approx \frac{10}{16}mV \approx 0.6mV$. Similarly, The value N for $V \approx 5mV$ is $N \approx 91$, giving $V_i \approx \frac{5}{9.6}mV \approx 0.52mV$. These values compare favorably with the sample temperature $T = 4.11K$.

We were not able to measure noise in the $2\mu m$ samples using bias that is strictly in the linear response regime ($eV_i \ll k_B T$). In fact, deviations from linear response in the range of bias used here are reflected in the sample conductance itself (Fig.5). The smallest bias used for the sample in Fig.6 was $V = 1.4mV$ which, in terms of V_i, is the equivalent of $3K$ (using the respective $N \simeq 29$ from Fig.8. This bias is not much smaller than the bath temperature. Nevertheless, the low bias regime we did manage to use is low enough to expose the peak in $Int(S_2)$ vs. V consistent with the proposed mechanism.

Note that the relevance of both correlated-noise scenarios discussed here hinges on specific assumptions. The DCR assumes that the conductance swings associated with slow fluctuators are potent enough to give a significant contribution. The non-linear mechanism we offered requires the existence of 'soft' fluctuators that, in addition, are coupled effectively to critical resistors. In either case the 'master fluctuator(s)' should operate on the frequency window that is relevant for the experimental scales. In the regime of mesoscopic samples one may expect to find fluctuators realizations such that some (or all) of these conditions are not realized in which case the non-Gaussianity will be weak or absent. Moreover, applying a voltage may displace a key fluctuator out of its 'commanding' position thereby removing the origin of correlations. This is certainly a concern in the strongly non-linear regime. Applying a large field will inevitably modify the current-carrying network. The modification may be reversible, in which case we expect that data such as in figures 6 and 7 will reproduce themselves under V cycling. However, when the applied V is sufficiently large, it is quite likely that a different percolation network will be precipitated, just as a thermal cycling involving high temperatures would cause [23]. This may lead to noise data of a different statistical nature. The change may well be subtle; turning on or off the cou-

pling of certain fluctuators to the CCN is all that is needed. The average disorder may not change in the V - recycling process, and the CF pattern may be only slightly affected. We believe that these considerations account qualitatively for the non-systematic occurrences of non-Gaussian noise in our samples, as well as for the effect of V cycling discussed in section II.

It should be mentioned that there are other non-linear mechanisms, not considered here, that may contribute to correlations between remote fluctuators, especially at the high V regime. For example, the change of the local voltages could modify the values of the critical resistors in the CCN [24], and may give rise to new fluctuators. Another interesting scenario is a heterodyne effect; namely, frequency mixing of the 'master' frequency with the 'local' one due to the non-linearity of the critical resistors. All such mechanisms, as well as the DCR, should be seriously considered whenever a noise with colored second-spectrum is encountered in a conducting system. Note that a generic feature of these mechanisms is their long range $1/R$ nature (see eq.2) making them more effective than most 'direct' interactions. As was demonstrated in this work, these effects may bring about correlations between fluctuators even in samples that are considerably larger than the scale relevant for the interactions associated with the electron-glass [23].

Finally, it should be mentioned that while reducing the applied V below the value where $Int(S_2)$ peaks diminishes the non-Gaussian effect, the glassy effects if anything, become more prominent [23, 25]. Clearly then, *glassiness and correlated-noise (when exists) are not necessarily related.*

In summary, we have described a set of conductance-noise experiments on disordered films of In_2O_{3-x} in their glassy phase. The emphasis in this study was on the degree of noise correlation as function of system size. Noise correlation was measured by the second spectrum of conductance data taken at liquid helium temperatures. Our main finding is that, down to sample size of $2\mu m$ the noise has the usual $1/f$ power-spectrum. Hopping samples with this size contain small number of critical resistors as indicated by the prominent conductance-fluctuations they exhibit. Such samples still show essentially all the electron-glass features as macroscopic samples [23]. Given the way non-Gaussianity decreases below a certain bias in these samples it seems unlikely that the correlations observed at finite bias are due to glassiness.

When the voltage used in the conductance measurements was not small enough, non-

Gaussian noise was observed in several of the samples, including samples as large as $40\mu m$. We have demonstrated that the degree of non-Gaussianity is a non-trivial function of the bias. A model that purports to account for these findings was offered, and its consequences are found to be in qualitative agreement with our experiments.

A lesson that may be taken from our study is that correlations in conductance noise may arise from a non-linear mechanism, and this may be of particular relevance to disordered conductors measured at low temperature. Such effects need be better understood and carefully examined before a non-Gaussian noise is associated with correlations due to, e.g., glass. As a point of principle, one indeed expects that, on sufficiently small scales, a glassy system may show a correlated noise. What should perhaps be stressed is that the converse is not necessarily true.

IV. APPENDIX

Here we consider the coupling between the fluctuators in more detail, and estimate the effect of this coupling on the second spectrum. For simplicity we assume that the (initially un-correlated) 'fast' fluctuators are coupled to 'slow' ones, and the coupling is characterized by the occupation numbers $\tilde{n}_j$. By 'fast' and 'slow' fluctuators we refer to specific two-level-systems that contribute in the measured noise, (at the high end of the fluctuators vs. frequency distribution and at the lower end respectively). Explicitly we consider only the modulation effect of 'slow' fluctuators on 'fast' one. The complementary process (i.e., 'fast' affecting 'slow') while possible is much more involved and is not treated here.

The 'slow' fluctuators affect the value of Δ_i of the 'fast' fluctuators. A natural result of this modification is change of the fluctuators occupation numbers, n_i.

Making use of Eq.1, and using $n = (\exp(\Delta/k_B T) + 1)^{-1}$ one concludes that in a presence of 'slow' fluctuator j one has

$$n_i(1 - n_i)|_t = n_i(1 - n_i)|_0 (1 - \frac{\tanh(\Delta_i/2k_B T)}{2k_B T}\tilde{n}_j(t)\delta\Delta_{i,j}) \tag{15}$$

Another source of the fluctuations is related to fluctuations of the relaxation time τ_i. One readily obtains:

$$\delta \left(\frac{\tau_i}{1 + (\omega \tau_i)^2} \right) = \delta(\tau_i) \frac{1 - (\omega \tau_i)^2}{(1 + (\omega \tau_i)^2)^2} \tag{16}$$

Thus, the contribution of $i - th$ resistor to the first spectrum is:

$$\delta \left((\delta n_i, \delta n_i)_\omega \right) = A_{ij} \tilde{n}_j; \tag{17}$$

$$A_{ij} = -(\delta n_i, \delta n_i)_\omega|_0 \frac{\tanh(\Delta_i/2k_B T)}{2k_B T} \tilde{n}_j(t) B_i \delta V_{i,j} -$$

$$-\tau_i \frac{1 - (\omega \tau_i)^2}{(1 + (\omega \tau_i)^2)^2} n_i (1 - n_i) \left(\frac{\alpha}{\Delta_i} - \frac{n_i}{k_B T} \right) B_i \delta V_{ij}$$

As a result, the first spectrum appears to be dependent on the occupation number of the 'slow' fluctuators.

Now let us estimate the effect of the correlations on the second spectrum $S_2(\omega_2)$. By definition for S_2 we have

$$S_2(\omega_1, \omega_2) = G^{-4} \int_0^{t_{max}} d\tau e^{i\omega\tau} \int_0^{t_{max}} dt$$

$$\cdot \int_t^{t+t_0} dt' \int_{t'}^{t'+t_0} d\tau' \delta G(t' + \tau') \delta G(t') e^{i\tau'\omega_1}$$

$$\cdot \int_{t+\tau}^{t+\tau+t_0} dt'' \int_{t''}^{t''+t_0} d\tau'' \delta G(t'' + \tau'') \delta G(t'') e^{i\tau''\omega_1} \tag{18}$$

where in our case

$$\delta G = \sum_i g_i \delta n_i \tag{19}$$

Thus, the spectrum of the fluctuations is related to the temporal behavior of δn_i. Here g_i are the coefficients describing a contribution of i-th fluctuator to the conductance fluctuations. Let us first consider a case of statistically independent fluctuators. In this case

$$\delta G(t') \delta G(t' + \tau') = \sum_i g_i^2 (\delta n_i(t') \delta n(t' + \tau')) \tag{20}$$

The DCR mechanism results from the modulation of the coefficient g_i in Eq.19 by another ('slow') fluctuator j. The corresponding contribution to S_2 can be estimated as

$$S_{2,DCR} \propto \sum_{i,j} \gamma_{ij} (\delta n_i, \delta n_i)_{\omega_1}^2 (\delta \tilde{n}j, \delta \tilde{n}_j)_{\omega_2} =$$

$$\bar{P} k_B T f_2(\omega_1) \bar{\gamma}_{ij} \sum_j (\delta \tilde{n}j, \delta \tilde{n}_j)_{\omega_2} \tag{21}$$

In contrast, in our case we deal with real coupling between the fluctuators described by Eq.1 which exists only at finite V. Using Eq.17 one obtains a contribution to S_2 $\propto \sum_{ij} A_{ij}^2 (\delta\tilde{n}_j, \delta\tilde{n}_j)$. The most important term here is the one $\propto (\alpha/\Delta_i)^2$. Indeed, due to a presence of 'soft' fluctuators with $\Delta_i \to 0$ this term is divergent. The divergency has a clear cut-off at $\Delta_i \sim \delta\Delta_i = B_i V_{ij}$ since in our calculations we assumed the fluctuations of Δ_i to be small (note that here we assume $eV_i < k_B T$). Summing over the different fluctuators gives

$$\sum_i \frac{(\delta\Delta_i)^2}{(\Delta_i)^2} \simeq (B\delta V)^2 \int_{\delta\Delta} d(\Delta) P(\Delta)(\Delta)^{-2} = \bar{P} B \delta V \tag{22}$$

where we have assumed that the distribution $P(\Delta) = (\bar{P})$ is flat. Thus here the summation over different Δ is controlled by the lower limit of 'soft' fluctuators, and we obtain

$$S_{2,corr} \propto \alpha^2 \bar{P} \bar{B}_i \bar{\delta} V_{ij} f_1(\omega_1) \sum_j (\delta\tilde{n}j, \delta\tilde{n}_j)_{\omega_2} \tag{23}$$

Assuming that the 'slow' fluctuators have exponentially broad scatter of relaxation times, the contribution to the second spectrum is $S_{2,corr} \propto \omega_2^{-1}$ (at the frequency scale $t_0^{-1} > \omega_2 > t_{max}^{-1}$).

Note that the non-linear effect we propose will dominate over the DCR mechanism even for moderate bias $eV_i < k_B T$. The reason is that among different fluctuators there exist those with small Δ_i. For such fluctuators the relative modulation of the relaxation rate due to fluctuations of Δ_i as it is seen from Eq. 5 appears to be $\propto \delta\Delta_i/\Delta_i$. Since the corresponding contribution to S_2 is proportional to $(\delta\Delta_i/\Delta_i)^2$ it leads to a significant enhancement of a role of 'soft' fluctuators with small Δ_i as is indicated by eq. 22.

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V. FIGURES CAPTIONS

FIG. 1. Conductance noise power-spectrum measured under bias voltage $V = 3$mV. The lower frequency part of the spectrum represents the averaging over 16 Fourier transformed $G(t)$ runs. The higher frequency part of the power-spectrum was averaged over 1024 power-spectrum curves measured on the resistor $R = 1.01 M\Omega$, connected in series with the sample, employing HP35670A. The dashed line depicts $1/f^{1.08}$ dependence (a best fit to the data). Inset: typical $G(t)$ series used for calculation of the lower frequency part of the power-spectrum. Note the relatively low signal to noise ratio typical for mesoscopic samples. The sample: length=$2\mu m$, width=$2\mu m$, $R_\square = 5M\Omega$.

FIG. 2. The magnitude of the noise power-spectrum measured at $f = 10mHz$ as a function of sample area A (= LxW). Each datum point represents the averaging over several samples with same area and values of the resistance ranging between $R_\square = 2M\Omega$ and $R_\square = 10M\Omega$. The dashed line depicts the $S_1 \propto A^{-1}$ law.

FIG. 3. Second-spectrum as a function of normalized frequency measured in two samples with the same lateral dimensions; $L = 30\mu m$, and $W = 40\mu m$, and resistances; $R_\square = 4.8M\Omega$, and $R_\square = 5.3M\Omega$ for samples in (a) and (b) respectively. The octaves (2 to 9) are labelled by the corresponding lowest frequency values (f_L) and represented by different symbols on the plot. The measurements for both samples were performed using a series resistor $R = 100k\Omega$ and applying $V = 100$mV, and $V = 150$mV as a bias voltage for the sample in (a) and (b) respectively. The degree of non-Gaussianity (labeled in this work as $Int(S_2)$, see text) is taken to be proportional to the area defined by the dashed and dotted lines, and the ordinate axis (plate a).

FIG. 4. The 'memory' cusp (see reference 9) as seen in the measurements of conductance as a function of the gate voltage for the sample shown in Fig.3(b). Two successively measured traces (solid and open circles) show reproducible conductance fluctuation (CF). Gate voltage scan rate was $0.02V/sec$, bias voltage $V = 20$mV.

FIG. 5. Conductance as a function of the applied bias measured for two samples with the same lateral dimensions; $L = 2\mu m$, $W = 2\mu m$. Note the deviation from linear response even at the smallest bias used.

FIG. 6. The degree of non-Gaussianity $Int(S_2)$ as function of the applied bias for two series of measurements: prior to (solid circles), and after (open squares) the application of high bias (see text). Dashed lines are guides for the eye. The measurements were performed using a series resistor $R = 1.01 M\Omega$. The sample: $L = 2\mu m$, $W = 2\mu m$, $R_\square = 5M\Omega$.

FIG. 7. Plate **a**: The degree of non-Gaussianity as function of the applied bias in two series of measurements: successively increasing the bias (open circles), and a later set, employing bias values within the same range as before with no particular order (solid circles). The value of η (in the first-spectrum law, $1/f^\eta$), and the noise power per decade as function of the applied bias - **(b)** and **(c)**, respectively. Dashed lines are guides for the eye. The measurements were performed using a series resistor $R = 1.01M\Omega$. Sample parameters: $L = 2\mu m$, $W = 2\mu m$, $R_\square = 9M\Omega$.

FIG. 8. Plate **(a)**: Conductance versus gate-voltage scans taken with different values of bias V for the sample in Fig.6. Each trace shows reproducible conductance fluctuations with rms amplitude that decreases with V. The average value of the conductance increases with bias (c.f., Fig.5). Plate **(b)**: The values of N for the series of traces shown in **(a)** estimated by the rms amplitude of the fluctuations (see text).

FIG. 9. A schematic dependence of $S_{2,corr}$ and $S_{2,DCR}$ on the applied voltage (see text).

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